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## Facilitator Information

## Learning Objectives (for content \& process)

After completing this activity, learners should be able to:

- Explain the pros \& cons of various simple search algorithms.
- Explain common tradeoffs between the complexity and performance of algorithms.
- Assess the performance of simple algorithms as a function of their input size N .

This activity should help learners develop teamwork and critical thinking skills.

## Prerequisites (for content \& process)

Before starting this activity, learners should have:

- Previous experience with POGIL (useful but not necessary).
- Programming experience is not required for the activity, but is for some applications.


## Preparation

1. Print 1 activity per 2-3 students; print worksheet and graph on separate pages (not 2-sided)
2. Optional: Provide the worksheet on the board, a poster, or in presentation software, so teams can see each others' work easily.
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## Activity Notes

- If students are unfamiliar with POGIL, it may be simpler to have teams of 3 (no Reflector).
- The facilitator should spend a minute or two introducing the activity.

While teams work, the facilitator should circulate among the teams to monitor progress and help with problems, but should try not to provide or confirm answers to key questions. For more information, see http://cspogil.org or http://pogil.org

- II. Player Strategies
- II.1. Report out: describe an algorithm. Summarize in table for class.
- II.2-4. Report out: rankings. Summarize in table for class.
- II.5. Mini-lecture: Describe specific examples where O() performance changes.
- III. Worst \& Average Case
- III.1. Some teams may get stuck on random guessing. Have everyone write down 1-2 random numbers, then list on board. Notice: duplicates; min \& max; multiples of 5, 10; odd vs. even; etc. (see Random Number Distribution Table for example)
- III.2. Report out: worst \& average case results. Summarize in table for class.
- IV. Parts of this may be too complex for some CSO/CS1 students.


## Things to Do

- ? Build histograms to better understand average \& worst case (see right)
- Add more guidance on mean \& max counts
- Add sections on: implications for search: effort required to sort, tradeoffs.
- Add applications, resources, rubrics
- Add summary section.
- Add supporting resources.


## Activity History

| \# of <br> guesses | possible <br> values guessed | \# of <br> values | total \# <br> values |
| :--- | :---: | ---: | ---: |
| 1 | 50 | 1 | 1 |
| 2 | 25,75 | 2 | 3 |
| 3 | $12,37,62,87$ | 4 | 7 |
| 4 | $6 \ldots 94$ | 8 | 15 |
| 5 | $3 \ldots 97$ | 16 | 31 |
| 6 | $2 \ldots 99$ | 32 | 63 |
| 7 | $1 \ldots 100$ | 34 | 100 |

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2012-12 revised
2013-08 revised based on experiences in India
2014-01 reviewed and endorsed by The POGIL Project

## Introduction

start
time:

In computing, we often must search in a set for a particular item. As computer scientists, we are particularly interested in searching very large sets, with thousands or millions of values. For example, the Harvard University Library has roughly 16,000,000 volumes, and the US Library of Congress has roughly 22 million cataloged books, and over 100,000,000 total items. In this activity, we use a simple game to explore some basic searching algorithms. This will also help us explore more general concepts in algorithm design and analysis, so studying searching is useful even though very few of us may need to implement searching algorithms, since efficient techniques are part of most software libraries (APIs).

Before you start, complete the form below to assign a role to each member.
If you have 3 people, combine Speaker \& Reflector.

| Team | Date |
| :--- | :--- |

## Reminders:

1. Note the time whenever your team starts a new section or question.
2. Write legibly \& neatly so that everyone can read \& understand your responses.

## I. (6 min) Hi-Lo Game

start
time:
$\mathrm{Hi}-\mathrm{Lo}$ is a number guessing game with simple rules, played by school children.
a. There are two players - A and B.
b. Player A thinks of a number from 1 to 100 .
c. Player $B$ guesses a number.
d. Player A responds with
"too high", "too low", or "you win".
e. Players B and A continue to guess \& respond until B wins (or gives up).


1. (1 min) How many different responses can player A give? $\qquad$
2. (1 min) When does the game end?
3. (2 min) Play the game a few times to ensure that everyone understands the rules.
4. ( 2 min ) Optional: List up to 3 ways to clarify the rules.

## II. (12 min) Player Strategies

start time:

1. ( 3 min ) Describe 4-5 different strategies that Player B could use to guess numbers.

Try to have a mixture of simple and clever strategies - in computer science, we call these algorithms.
For example, one strategy is "Count upwards, starting from 1."
Name each strategy and list it in the first column of the Hi-Lo Game: Worksheet.
Before you continue, review progress with the facilitator.
2. ( 2 min ) Evaluate each algorithm (strategy) with regard to how quickly it will find the right answer, by ranking from 1 (fewest guesses) to 5 (most guesses).
Add the rankings to the worksheet in a column labeled Quick.
3. (2 min ) Evaluate each algorithm with regard to how easy it is to describe or specify, by ranking from 1 (easiest) to 5 (hardest).
(Suppose you had to explain each algorithm to a first-grader so that she could play the game.)
Add the ranking to the worksheet in a column labeled Easy.
4. (1 min) For each algorithm, plot its Quick and Easy values on the Hi Lo Game: Graph.
5. ( 3 min ) In complete sentences, describe the relationships between the Quick and Easy rankings, including what you see from the graph. Before you continue, review progress with the facilitator.

## III. (10 min) Worst \& Average Case Performance

start time:

1. ( 3 min ) We could compare the algorithms' speeds using the number of guesses.

For each algorithm, determine the worst case (maximum) number of guesses required to win.
Add the numbers to the worksheet in a column labeled Worst.
2. (3 min) For each algorithm, determine the average case (typical) number of guesses required to win. Add the numbers to the worksheet in a column labeled Average.
Note that the minimum number of guesses is always 1 - it's nice to be lucky.
Hint: there is a pattern between worst and average cases, but it does not apply for all strategies.
3. ( 2 min ) List 3 reasons why the number of guesses could be a better way to compare algorithms. Before you continue, review progress with the facilitator.

## IV. (10 min) Effect of Input Size

start
time:

1. ( 3 min ) Assume that Player A chooses a number from 1 to 1000
(instead of 1 to 100). For each algorithm, what are the worst case \& average case number of guesses? Add the numbers to the worksheet in columns labeled " 1 K Worst" and " 1 K Average".
2. (4 min ) Optional: Assume that Player A chooses a number from 1 to N .
(For example, $\mathrm{N}=100, \mathrm{~N}=1000, \mathrm{~N}=1,000,000$ )
For each algorithm, what are the worst case \& average case number of guesses in terms of N ? Add the expressions to the worksheet in columns labeled " N Worst" and " N Average".
(Hint: you've already done $\mathrm{N}=100$ and $\mathrm{N}=1000$; consider other values before generalizing to N .)
3. (3 min) Describe the pros \& cons of analyzing performance in terms of input size N .

## Applications

1. Team Meeting Minutes (TMM) - see common description (separate doc) and rubric (below).
2. Personal Reflection Memo (PRM) - see common description (separate doc).
3. Write a program (or set of programs) to play the game using each algorithm.

The program should count the guesses and print the total when the game ends.
Record how much time it takes to code each algorithm. Is this consistent with your "Easy" rating?
Play the game 5 times with each algorithm. Is the guess count consistent with your "Quick" rating?

## Team Meeting Minutes (TMM): Rubric

| SPECIFIC CRITERIA | RATING | COMMENTS |
| :---: | :---: | :---: |
| I: Comprehensively synthesizes guessing algorithms. | / 4 |  |
| II: Comprehensively synthesizes rankings \& tradeoffs. | / 4 |  |
| III: Comprehensively synthesizes worst \& average case performance. | / 3 |  |
| IV: Comprehensively synthesizes N -analysis. | / 3 |  |
| COMMON CRITERIA | RATING | COMMENTS |
| Begins with a summary of the activity; ends with a summary of questions, and a list of action items, if needed. | / 3 |  |
| Mechanics \& format: <br> Work is neat, well organized, \& clearly provides all required information, including team member roles \& timing data. | / 3 |  |
| TOTAL | / 20 |  |

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## Facilitator - Answer Key

## I. (6 min) Hi-Lo Game

1. There are 3 possible answers - too high, too low, you win.
2. The game ends when B wins or gives up.

## II. (12 min) Player Strategies

1-4. See worksheet answer key below.
5. In general, simple algorithms tend to be slower, and faster algorithms tend to be more complex.

## III. (10 min) Worst \& Average Case Performance

1. Measuring time with a stopwatch is objective \& quantifiable, but may be affected by other factors math aptitude of the person playing the game, reaction time of the timer.
2-3. See worksheet key below.
2. Number of guesses is a better measure because it depends less on other factors.

## IV. (10 min) Effect of Input Size

1-2. See worksheet key below.
3. Pros \& cons of analyzing performance in terms of input size $N$.

Pros: generalizes from specific examples
Cons: more difficult calculations \& logic.
If teams have questions about random guessing, have each student write down 1-2 random numbers, then list or plot for entire class. Typical outcomes: no multiples of 10 or 5, more odds than evens

| 90 |  |  |  |  |  |  |  |  |  | $X$ |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 80 |  |  |  | X |  |  |  |  |  |  |
| 70 |  |  | X | X | X |  |  |  |  |  |
| 60 |  |  |  |  | X |  |  |  |  |  |
| 50 |  |  | X |  |  |  |  | X |  |  |
| 40 |  |  | XX | X |  | X |  | X |  |  |
| 30 |  |  |  |  |  |  | X |  |  |  |
| 20 |  |  | X |  |  |  |  | X |  |  |
| 10 |  | X | X |  | XX |  |  | X |  | X |
| 0 |  |  |  | XX |  |  |  | XX |  | XX |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

## Hi-Lo Game Worksheet - Answer Key

| \# | Player Algorithm |  |  |  |  |  |  |  |  | $\begin{aligned} & \dot{y} \\ & \stackrel{y}{⿺} \\ & z \\ & \dot{m} \\ & \geq \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | Guess numbers at random, ignoring "hi-lo" feedback, without memory (repeat guesses possible) | 5 | 2 | 10 | 1K | 50 | 10K | 500 | 10N | N/2 |
| B | Guess numbers at random, ignoring "hi-lo" feedback, but don't repeat guesses | 4 | 3 | 12 | 100 | 50 | 1000 | 500 | N | N/2 |
| C | Count up from 1 (or down from 100) | 3 | 1 | 3 | 100 | 50 | 1000 | 500 | N | N/2 |
| D | Count up by 10s, then down by 1s (many variations) | 2 | 4 | 8 | 20 | 10 | $\begin{aligned} & \hline 110 \\ & (30) \end{aligned}$ | $\begin{array}{r} 55 \\ (15) \end{array}$ |  |  |
| E | Guess 50, then 25 or 75, then 12 or 38 or 62 or 88 - keep track of min \& max possible values, divide range in half | 1 | 5 | 5 | 7 | 6 | 11 | 10 | $\log _{2} \mathrm{~N}$ | $\log _{2} \mathrm{~N}$ |

## Hi-Lo Game Worksheet

| 71 | $\pi$ | 0 | n | $\boldsymbol{\infty}$ | > | \# |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\begin{array}{\|l} \hline D \\ 09 \\ 0 \\ 2 \\ \frac{1}{2} \\ \frac{3}{3} \\ 2 \\ 0 \\ 0 \\ 3 \\ 0 \end{array}$ |
|  |  |  |  |  |  | Quick |
|  |  |  |  |  |  | Easy |
|  |  |  |  |  |  | Worst |
|  |  |  |  |  |  | Average |
|  |  |  |  |  |  | 1K <br> Worst |
|  |  |  |  |  |  | 1K <br> Average |
|  |  |  |  |  |  | N <br> Worst |
|  |  |  |  |  |  | N <br> Average |

## Hi-Lo Game: Graph



## Random Number Distribution

| 90 |  |  |  |  |  |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 80 |  |  |  |  |  |  |  |  |  |  |
| 70 |  |  |  |  |  |  |  |  |  |  |
| 60 |  |  |  |  |  |  |  |  |  |  |
| 50 |  |  |  |  |  |  |  |  |  |  |
| 40 |  |  |  |  |  |  |  |  |  |  |
| 30 |  |  |  |  |  |  |  |  |  |  |
| 20 |  |  |  |  |  |  |  |  |  |  |
| 10 |  |  |  |  |  |  |  |  |  |  |
| 0 |  |  |  |  |  |  |  |  |  |  |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

